MTHS24 – Exercise sheet 10

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Thurday, 25 July 2024

Lecture material

Discussed topics:

- Two-particles quantization condition
- Finite-folume spectrum
- Interacting energy levels from resonances

References:

- Original Lüscher paper (stable particles): inspire - Lüscher (1985)
- Original Lüscher paper (scattering): inspire - Lüscher (1986)
- Non-rest frames: inspire - Rummukainen & Gottlieb (1995)
- Field theory approach: inspire - Kim, Sharpe, & Sachrajda (2005)
- Coupled-channels: inspire - Hansen & Sharpe (2012)
- Arbitrary number of channels and spin: inspire - Briceno (2014)

Exercices

10.1 Two Particles in a Box

In this problem, we explore the discrete spectrum of two particles in a finite cubic volume and its relation to their infinite volume scattering amplitude.

(a) Enumerate the different momenta (in units of $2\pi/L$) allowed for $\mathbf{n}^2 \in \{0, 1, 2, 3, 4, 5, 6, 8, 9\}$.

(b) Take the limit of E/m as $mL \to \infty$.

(c) Plot the non-interacting spectrum in terms of $E^\star/m$ in the rest frame $\mathbf{n}_P = [000]$ as a function of mL.

(d) Repeat (c) for the frames $\mathbf{n}_P = [001], [011], [111], [002]$.

10.2 Finite-Volume Function

This problem focuses on the finite-volume function $F(E, \mathbf{P}, L)$, which characterizes finite-volume distortions in an interacting system.

(a) Derive $F(E, \mathbf{P}, L)$ using the all-orders approach discussed in the lectures. Simplify the result for numerical computation.

(b) Determine the dimensions of F.

(c) For a system at total momentum $\mathbf{P} = \mathbf{0}$, plot F as a function of $E^\star/m$, $(Lq^\star/2\pi)$ for fixed $mL = 4, 5,$ and 6.

(d) Repeat (c) for moving frame systems, $\mathbf{n}_P = [001], [011], [111], [002]$.

10.3 Connecting the Finite-Volume Function to the Spectrum

In this problem we explore how to determine the spectrum of two non-interacting particles by solving $F^{-1} = 0$.

(a) Find the spectrum of two non-interacting particles in their rest frame by solving $F^{-1} = 0$ for fixed $mL = 4, 5$, and 6.

(b) Repeat for moving frame systems, $\mathbf{n}_P = [001], [011], [111], [002]$.

10.4 Lüscher Quantization Condition

Here, we study the poles of the correlation matrix for interacting particles using the Lüscher quantization condition.

(a) Show that the imaginary parts of $\mathcal{M}^{-1}$ and F cancel.

(b) Using the Breit-Wigner and Effective Range parameterizations, investigate the spectrum of an interacting two-particle system.