Ruhr-University Bochum | Modern Techniques in Hadron Physics

MTHS24 – Exercise sheet 2

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Lecture material

Discussed topics:

- Amplitude generalities
- Canonical and helicity states
- Analyticity
- Unitarity
- Crossing symmetry

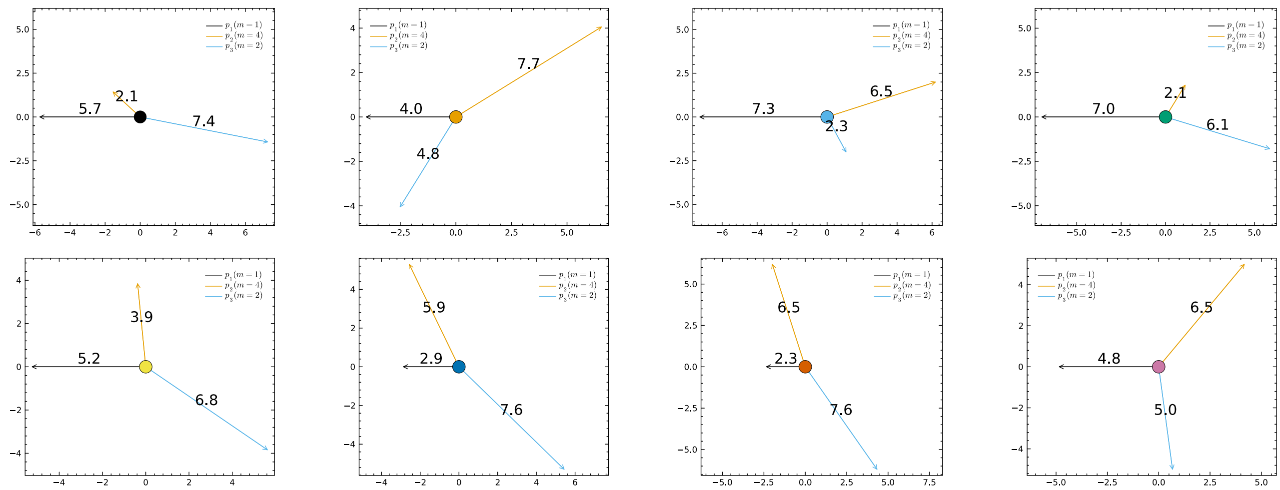
References:

- A.D. Martin, T.D. Spearman, Elementary Particle Theory, inSpire
- Eero Byckling, K. Kajantie, Particle Kinematics, inSpire

Exercices

2.1 Decay kinematics

Relate the values of momenta and their orientation to location on the Dalitz plot for the decay $p_0 \to p_1 + p_2 + p_3$ using values indicated on the figures below.

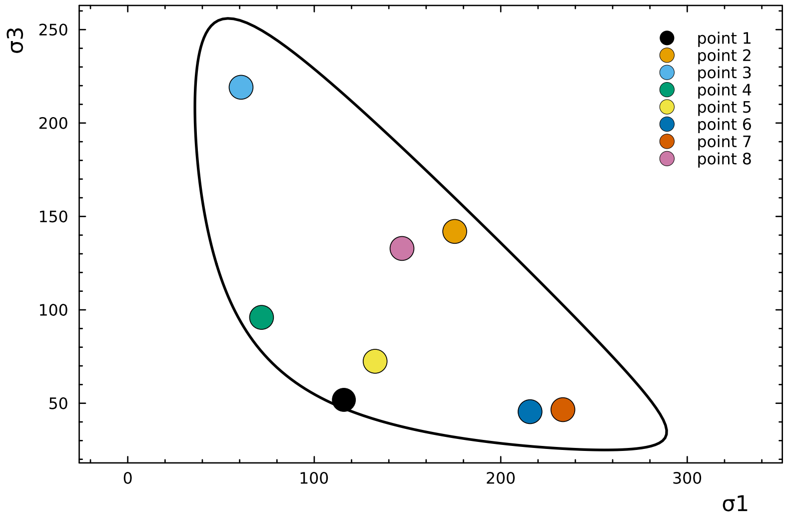


Solution: Mass is computed as

$$\begin{aligned} m_{ij} = (p_i + p_j)^2 &= m_i^2 + m_j^2 + 2E_iE_j - 2p_ip_j \cos\angle\vec{p}_i\vec{p}_j \\ &= m_i^2 + m_j^2 + 2E_iE_j + p_i^2 + p_j^2 - p_k^2 \\ &= (E_i + E_j)^2 - p_k^2 \end{aligned}$$

the angle between the two momenta is traded to magnitude of momenta using algebra of three-vectors, $-\vec{p}_k = \vec{p}_i + \vec{p}_j$

$$p_k^2 = p_i^2 + p_j^2 + 2p_ip_j \cos\angle\vec{p}_i\vec{p}_j$$



2.2 2-to-2 Kinematics

In hadron physics, the Mandelstam invariants s, t, and u are essential for describing the kinematics of scattering processes. Consider a $2 \to 2$ scattering process where two particles with four-momenta p_1 and p_2 scatter into two particles with four-momenta p_3 and p_4. Consider the particles have different masses. The Mandelstam invariants are defined as $s = (p_1 + p_2)^2$, $t = (p_1 - p_3)^2$, and $u = (p_1 - p_4)^2$. The scattering angle θ is the angle between the momenta of the incoming ($\vec{p}_1$) and outgoing ($\vec{p}_3$) particle in the center of mass frame.

(a) Verify the relation $s + t + u = \sum m_i^2$, where m_i are the masses of the particles.

(b) Calculate the particle energies and 3-momenta in the center of mass frame, as a function of s

(c) Calculate the scattering angle θ in terms of s, t, and u

(d) How would those relation change if one considered the crossed reaction $p_1p_{\bar{3}} \to p_{\bar{2}}p_4$?

2.3 Canonical and Helicity states

Consider a particle of spin $1/2$ at rest with spin up $u_+ = \sqrt{2m}\begin{pmatrix}1\\0\\0\\0\end{pmatrix}$ or spin down, $u_- = \sqrt{2m}\begin{pmatrix}0\\1\\0\\0\end{pmatrix}$.

Remember that the generator of Lorentz group is $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$. That implies that

$$R_y(\theta) = \exp\left(-i\frac{\theta}{2}\sigma_y\right) = \begin{pmatrix} \cos\frac{\theta}{2} - i\sigma_y \sin\frac{\theta}{2} & 0 \\ 0 & \cos\frac{\theta}{2} - i\sigma_y \sin\frac{\theta}{2} \end{pmatrix} \tag{1}$$

$$B(0 \to \vec{p}) = \exp\left(\frac{\vec{\eta}}{2} \cdot \vec{\alpha}\right) = \begin{pmatrix} \cosh\frac{\eta}{2} & \hat{\eta}\cdot\vec{\sigma}\sinh\frac{\eta}{2} \\ \hat{\eta}\cdot\vec{\sigma}\sinh\frac{\eta}{2} & \cosh\frac{\eta}{2} \end{pmatrix} \tag{2}$$

where $\tanh\eta = p/E$, η being the particly rapidity.

(a) Derive the spinors in canonical and helicity basis for a particle with momentum lying in the xz plane.

2.4 Boosting spinning particles

A particle of spin 1 in the helicity basis lying in the xz plane is described by the following polarization vectors:

$$\epsilon^{\mu}_{\pm 1}(\theta) = \left(0, \mp\frac{\cos\theta}{\sqrt{2}}, -\frac{i}{\sqrt{2}}, \pm\frac{\sin\theta}{\sqrt{2}}\right) \tag{3}$$

$$\epsilon^{\mu}_{0}(\theta) = \left(\frac{p}{m}, \sin\theta\frac{E}{m}, 0, \cos\theta\frac{E}{m}\right) \tag{4}$$

$$p^{\mu}(\theta) = (E, \sin\theta p, 0, \cos\theta p) \tag{5}$$

Consider a particle with mass m and momenum $p = m\hat{z}$ and with helicity $+1$.

(a) Boost the momentum and the polarization in the x direction of a boost $\beta = 1/\sqrt{2}$.

(b) Compare with the polarization vectors according to the new momentum. Decompose it in the basis of the new polarization vectors.