# MTHS24 – Exercise sheet 3

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## Lecture material

**Discussed topics**:

- Partial waves
- Analyticity
- Unitarity

**References**:

- A.D. Martin, T.D. Spearman, Elementary Particle Theory, inSpire
- Review on Novel approaches in hadron spectroscopy by JPAC, inspire

## Exercices

### 3.1 Amplitude analysis

Given the Omnès function:

$$\Omega(s) = \exp\left[\frac{s}{\pi}\int_{4m_\pi^2}^{\infty} dz \frac{\delta(z)}{z(z-s)}\right], \tag{1}$$

(a) Consider the low-energy expansion of the pion form factor $F_\pi(s) \equiv \Omega(s)$:

$$F_\pi(s) = 1 + \frac{1}{6}\langle r_\pi^2\rangle\, s + \mathcal{O}(s^2), \tag{2}$$

and deduce the sum rule:

$$\langle r_\pi^2\rangle = \frac{6}{\pi}\int_{4m_\pi^2}^{\infty} dz \frac{\delta(z)}{z^2}. \tag{3}$$

The quantity $\sqrt{\langle r_\pi^2\rangle}$ is called charge radius of the pion, see PDF for summary of the experimental measurements.

(b) Assume that the phase shift $\delta(s)$ reaches $k\pi$ ($k$ is an integer) at $s = \Lambda^2$ and stays at that value for larger $s$. What is the behavior of $\Omega(s)$ in the limit $|s| \to \infty$?

(c) What is the resulting function $\Omega(s)$ for an infinitely narrow resonance, *i.e.* consider $\delta(s) = \pi\theta(s - M^2)$?

### 3.2 Isospin

The one-pion states are of the form $|I, I_3\rangle$ with

$$|1,+1\rangle = |\pi^+\rangle, \quad |1,0\rangle = |\pi^0\rangle, \quad |1,-1\rangle = |\pi^-\rangle. \tag{4}$$

(a) From the three pion flavors, construct the nine different two-pion states and their decomposition into isospin states.

(b) Invert the decomposition in (a) to get the decomposition of the isospin states into two pion states.

(c) Do pions obey Bose statistics? What is the angular momentum (even or odd) for the states with $I = 0, 1, 2$?