MTHS24 – Exercise sheet 5

Morning: Eric Swanson / Mikhail Mikhasenko
Afternoon: Nadine Hammoud, Dhruvanshu Parmar

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Lecture material

Discussed topics:

- Three-body decay kinematics
- Cascade parametrization of decays
- Helicity and covariant formalism

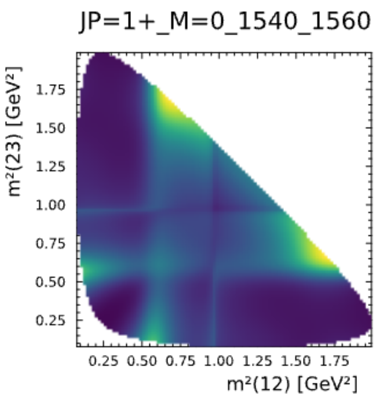
References:

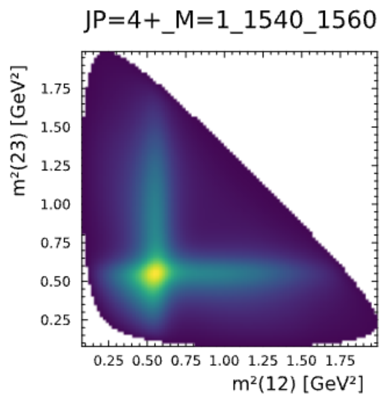
- A.D. Martin, T.D. Spearman, Elementary Particle Theory, inSpire
- Eero Byckling, K. Kajantie, Particle Kinematics, inSpire

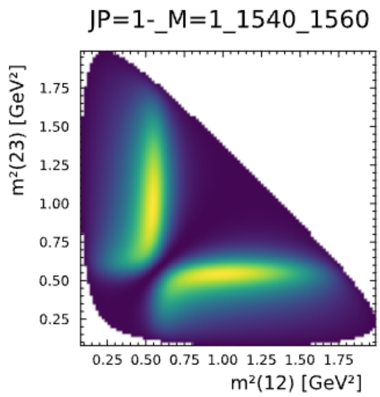
Exercices

5.1 Projections of the Dalitz Plot

For a given Dalitz plot, sketch the projections onto m_{12}^2, m_{23}^2, m_{31}^2, and helicity angles for all subsystems rest frames.

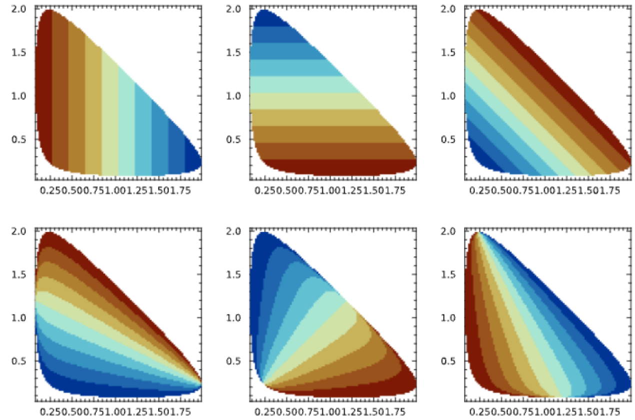






Solution:

What do I integrate



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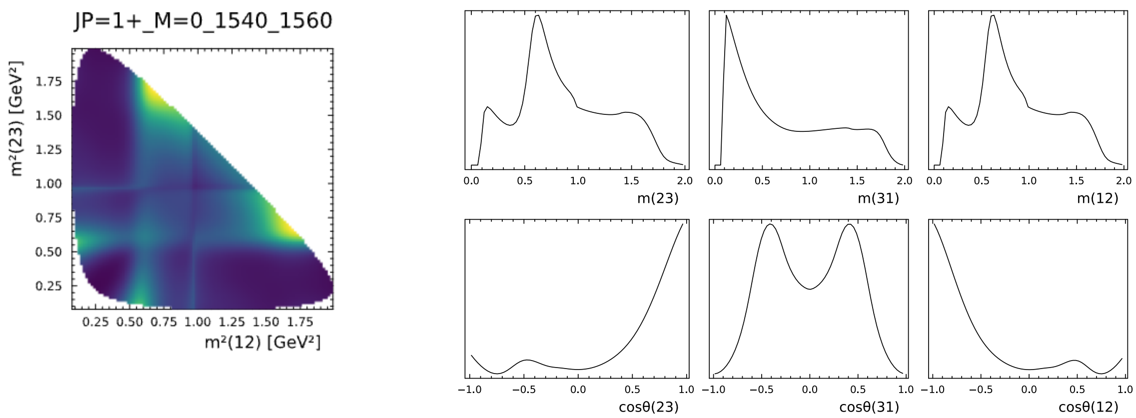

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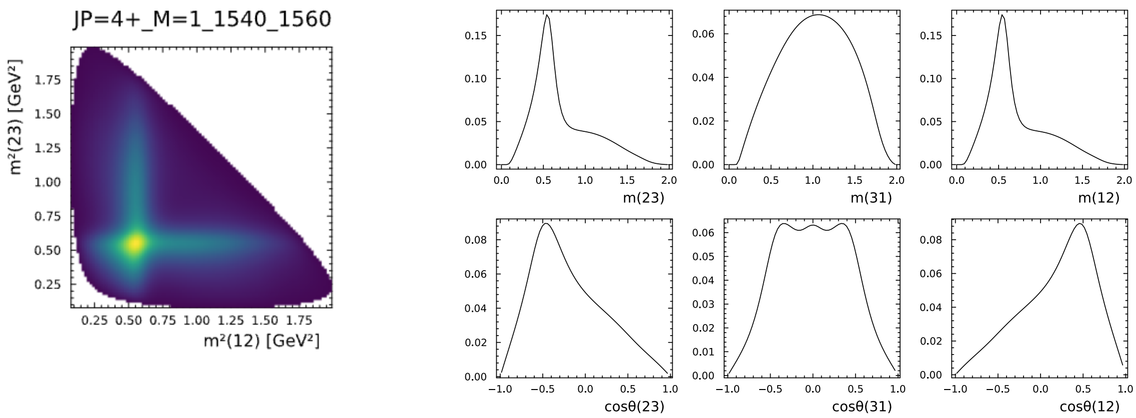

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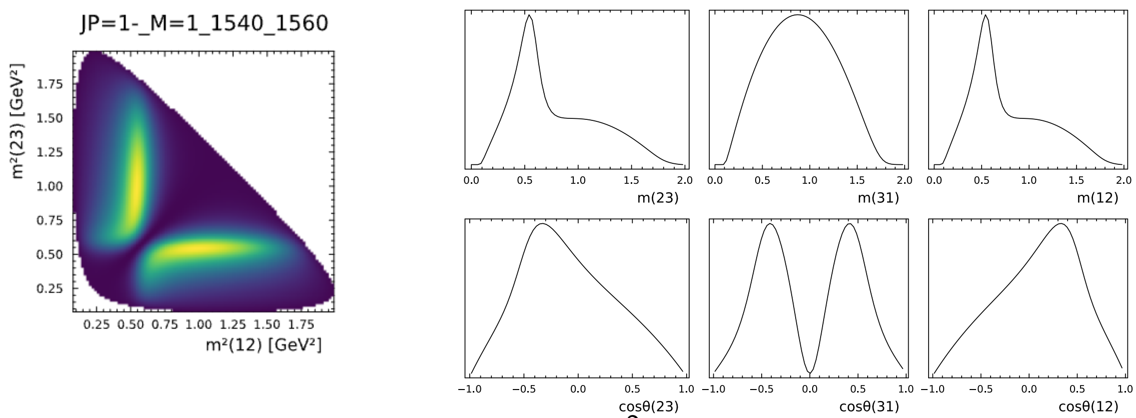

5.2 Spin Sum

Polarization vectors of a spin-one particle are given by

$$\epsilon^{\mu}_{\pm 1}(\theta) = \left(0, \mp\frac{\cos\theta}{\sqrt{2}}, -\frac{i}{\sqrt{2}}, \pm\frac{\sin\theta}{\sqrt{2}}\right) \tag{1}$$

$$\epsilon^{\mu}_{0}(\theta) = \left(\frac{p}{m}, \frac{E}{m}\sin\theta, 0, \frac{E}{m}\cos\theta\right) \tag{2}$$

(a) Evaluate the amplitude for the decay of a_1 meson ($J^P = 1^+$) to $\rho\pi$ using the covariant expression, $\mathcal{M} = \epsilon_{a_1} \cdot \epsilon_\rho$ in the following frames of reference:

- a_1 is at rest and the decay particles are aligned along the z-axis,
- ρ is at rest and a_1 and π are aligned along the z-axis, compare to (a) .

> **Solution:** The first one is $-(1, \gamma_\rho, 1)$, the second one is $-(1, \gamma_{a_1}, 1)$. Both equal since the two frames are connected by a boost with $\gamma = (m_{a_1}^2 + m_\rho^2 - m_\pi^2)/(2 m_{a_1} m_\rho)$.

(b) Compare three matrix elements in (a) configurations to the expectations from the helicity formalism,

$$A^L_{\lambda_{a_1}\lambda_\rho} = H^L_{\lambda_\rho,0}\, d^1_{\lambda_{a_1},\lambda_\rho}(\theta)\,, \tag{3}$$

where the helicity coupling is parametrized in LS scheme reads,

$$H^L_{\lambda_\rho,0} = \langle L, 0; 1, \lambda_\rho | 1, \lambda_\rho\rangle\ . \tag{4}$$

Which partial waves are allowed in the decay, and what value of L the covariant matrix $\mathcal{M}$ element correspond to?

> **Solution:** The S-wave corresponds to $(1, 1, 1)$, while D-wave is $(1, -2, 1)/\sqrt{10}$. Convariant formalism mixes partial waves.

5.3 Spin of a New Λ_b^{**0} State

A new Λ_b^{**0} state has been discovered decaying into $\Lambda_b^0\pi^+\pi^-$ with a prominent Σ_b^* resonance line on the Dalitz plot. The decay intensity distribution along the Σ_b^* band is provided in the supplementary material, which includes the helicity angle distribution. Your task is to determine the spin J of the Λ_b^{**0} state.

(a) Write down the decay matrix element for $\Lambda_b^{**0} \to \Lambda_b^0\pi^+\pi^-$ using helicity formalism.

(b) Identify the partial waves in the decay $\Sigma_b^* \to \Lambda_b^0\pi$.

(c) Determine the partial waves in the decay $\Lambda_b^{**0} \to \Sigma_b^*\pi$ for $J^P = \frac{1}{2}^\pm, \frac{3}{2}^\pm, \frac{5}{2}^\pm$.

(d) Compute the unpolarized differential distribution given by:

$$\frac{\mathrm{d}I}{\mathrm{d}\cos\theta} = \sum_{\lambda_0,\lambda_1}^{\{-1/2,1/2\}} \left| \langle L, 0; 3/2, \lambda_0 | J, \lambda_0\rangle\, d^{3/2}_{\lambda_0,\lambda_1}(\theta)\, \langle 1, 0; 1/2, \lambda_1 | 3/2, \lambda_1\rangle \right|^2$$