Ruhr-University Bochum Modern Techniques in Hadron Physics

MTHS24 – Exercise sheet 9

Morning: Laura Tolos / Andrew Jackura
Afternoon: XXX, YYY

Wednesday, 24 July 2024

Lecture material

Discussed topics:

- Hyperon-nucleon and hyperon-hyperon interactions
- Meson-exchange models
- Chiral effective field theory
- Hyperons in matter
- Brueckner-Goldstone Theory: Brueckner-Hartree-Fock approach

References:

- R. Machleidt, Advances in Nuclear Physics **19**, 189 (1989)
- R. Machleidt, D. R. Entem, Phys. Rept. **503**, 1 (2011)
- A Pich Rep. Prog. Phys. **58**, 563 (1995)
- V. Koch, Int. J. Mod. Phys. E **6**, 203 (1997)
- S. Petschauer, J. Haidenbauer, N. Kaiser, U. G. Meißner and W. Weise, Front. in Phys. **8**, 12 (2020)
- B. D. Day, Reviews of Modern Physics **39**, 719 (1967); **50**, 495 (1978)
- R.D. Mattuck, A guide to Feynman Diagrams in the Many-Body problem, Dover, New York, 1992. Editor McGraw-Hill, Inc.

Exercices

9.1 Effecitve Theory Questions

(a) Explain the Yukawa's idea (for NN interaction)

(b) Explain what an effective theory is and indicate the four pillars where the power of an effective theory lies in.

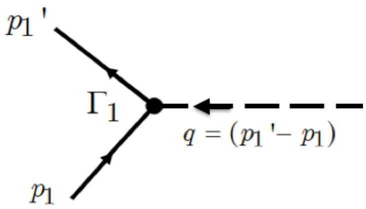
(c) Explain the many-body problem (for NN interaction)

9.2 Gradient Coupling

Consider the pseudo-vector (or gradient coupling) to the nucleon described by the Lagrangian

$$\mathcal{L} = -\frac{f_{\pi NN}}{m_\pi}\bar{\psi}\gamma^\mu\gamma_5\vec{\tau}\psi \cdot \partial_\mu \vec{\phi}^{(\pi)}, \tag{1}$$

and **compute the contribution of the following diagram to the one-pion exchange potential (OPEP)**



Some hints:

- You should compute $\bar{u}(p_1', s_1)\Gamma_{\pi NN} u(p_1, s_1)$, with $\Gamma_{\pi NN} = (i)^2 \frac{f_{\pi NN}}{m_\pi}\gamma^\mu \gamma_5 \vec{\tau} q_\mu$ for the incoming pion. Note that i is the imaginary unit, (γ^μ, γ_5) are the gamma matrices, $\vec{\tau}$ is the isospin vector, q is the four-momentum carried by the pion ($q_\mu = p_1' - p_1$), $f_{\pi NN}$ is the πNN coupling and m_π is the pion mass.
- Consider the static limit ($q_0 \to 0$)
- The Dirac spinors $u(p, s)$ in the non-relativistic approach are given by $u(p, s) = \begin{pmatrix} \chi_s \\ 0 \end{pmatrix}$, with χ_s the two-component Pauli spinor.
- The gamma matrices are defined as

$$\gamma^0 = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix}, \quad \gamma^k = \begin{pmatrix} 0 & \sigma^k \\ -\sigma^k & 0 \end{pmatrix}, \quad \gamma^5 = \gamma_5 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}$$

  with σ^k the three Pauli matrices (k running from 1 to 3). Also $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$ and $\{\gamma_5, \gamma^\mu\} = 0$, with μ and ν running from 0 to 3. The metric tensor is $g_{00} = +1$, $g_{kk} = -1$, $g_{\mu\neq\nu} = 0$.

9.3 Nucleon-nucleon potential with scalar meson exchange

Calculate the nucleon-nucleon potential due to scalar meson exchange. Things to be considered:

- you have to reproduce the expression on the slides;
- the scalar propagator is given by

$$\frac{i}{q^2 - m_s^2},$$

  where m_s is the scalar mass and q the four-momentum;
- work in the center-of-mass frame.
  If $\vec{p}_1$ y $\vec{p}_2$ are the momenta of initial particles 1 y 2, respectively, and $\vec{p'}_1$ y $\vec{p'}_2$ are the momenta of the final particles 1 y 2, respectively, then we can define

$$\begin{aligned} \vec{p}_1 &= -\vec{p}_2 = \vec{p}, \\ \vec{p'}_1 &= -\vec{p'}_2 = \vec{p'}. \end{aligned}$$

  With these definitions, we define

$$\begin{aligned} \vec{k} &= 1/2(\vec{p} + \vec{p'}), \\ \vec{q} &= \vec{p'} - \vec{p}; \end{aligned}$$

- work in the non-relativistic approximation: $E + M \sim 2M$;
- the angular momentum $\vec{L}$ is defined as $\vec{L} = \mathrm{i}\ (\vec{k} \times \vec{q})$.